Reduced total energy requirements for a modified Alcubierre warp drive spacetime

F. Loup
Royal Sun Alliance
Avenida Duque De Avila 141 2 Andar 1050 Lisbon Portugal*

D. Waite
Modern Relativity†

E. Halerewicz, Jr.
Lincoln Land Community College
5250 Shepherd Road, Springfield, IL 62794 USA‡

July 29, 2001

Abstract

It can be shown that negative energy requirements within the Alcubierre spacetime can be greatly reduced when one introduces a lapse function into the Einstein tensor. Thereby reducing the negative energy requirements of the warp drive spacetime arbitrarily as a function of $A(ct, \rho)$. With this function new quantum inequality restrictions are investigated in a general form. Finally a pseudo method for controlling a warp bubble at a velocity greater than that of light is presented.

1 Introduction

In recent years the possibility of interstellar travel within a human lifetime has become a hurtle for theoretical physics to over come. The discussion involves the use of radically transforming the geometry of spacetime to act as a global means of providing apparent Faster Than Light (FTL) travel. In 1994 Miguel Alcubierre then of Wales University introduced an arbitrary spacetime function which provided an apparent means of FTL travel within the frame work of General Relativity (GR) [1]. The drawbacks of this revolutionary new form of propulsion results from the production of causally disconnected spacetime regions and major violations of the standard energy conditions within GR. Further investigations into the warp drive spacetime have suggested that any manipulation of the spacetime coordinates in regards to FTL travel requires negative

*This address is given for mailing purposes only, although I hold a professional position at the above establishment my work was carried out independently and does not reflect RSA.; email: loupwarp@yahoo.com
†homepage: http://www.modernrelativity.com; email: FineS137@aol.com
‡Student of above institution only; email: ehj@warpnet.net

energy densities [2]. Alcubierre's generic model coupled with causally disconnected spacetime regions and large negative energy densities make it hard to build a justifiable model of a working warp drive spacetime. However recently it has been shown that quantum inequalities can allow for the existence of negative energy densities [3], and it has also been found that classical scalar fields can generate large fluxes of negative energy in comparison to Quantum Inequality (QI) restrictions [4]. Coupled with recent astrophysical and cosmological observations in support of negative energy in the form of Quintessence [5] we feel that there is still new life to be brought into the warp drive spacetime. The once distant dream of reaching the stars may become a reality with the aid of the warp drive spacetime using latin we can express our motto as Ex Somnium Ad Astra (ESAA) which translates; from a dream to the stars[1]. The following work was made possible as a collaboration of the ESAA motto in order to produce a physically justifiable model of the warp drive spacetime[2]. The inspiration behind this work was to choose a somewhat less arbitrary means of defining the *Alcubierre Warp Drive* spacetime. Specifically we have set out the task of redefining the functions of the warp drive spacetime in order to reduce the overall energy requirements as seen in respect to the Einstein tensor.

1.1 The Alcubierre spacetime

The Story of the Warp Drive begins with the Alcubierre paper [1], where we have the following metric:

$$ds^2 = dt^2 - (dx - v_s f(r_s) dt)^2 - dy^2 - dz^2. \tag{1}$$

We have also modified the metric signature in (1) to correspond to $(1, -1, -1, -1)$, for reasons which will become clear in later sections. The Alcubierre spacetime is further defined with the following functions

$$v_s(t) = \frac{dx_s(t)}{dt} \tag{2}$$

$$r_s(t) = [(x - x_s(t))^2 + y^2 + z^2]^{1/2}. \tag{3}$$

This spacetime model is often referred to a 'top hat' model, by means of a bump parameter σ, which is seen through

$$f(r_s) = \frac{\tanh(\sigma(r_s + R)) - \tanh(\sigma(r_s - R))}{2\tanh(\sigma R)} \tag{4}$$

This equation then allows for a "warp bubble" to develop in which a *star ship* may ride. The generalities of the Alcubierre top hat function can be described by the given volume expansion

$$\theta = v_s \frac{x_s}{r_s} \frac{df}{dr_s}. \tag{5}$$

[1]The authors would like to note that the ESAA motto was created by Simon Jenks who helped to formalize the early discussions regarding this work.

[2]The collaboration of this work was made possible through the online discussions forum; *Alcubierre Warp Drive* currently available at the following URL: http://clubs.yahoo.com/clubs/alcubierrewarpdrive

This is where we receive our first scientific definition of a 'warp drive.' *The volume elements expand behind the spaceship, while contracting in front of it.* The draw back of this work occurs with equation 19, where the energy density $T^{ab}n_a n_b$ violates the weak, strong, and dominate energy conditions as seen by an Eulerian observer [1]:

$$\rho = T^{ab}n_a n_b = a^2\, T^{00} = \frac{c^2}{8\pi G}\, G^{00} = -\frac{c^2}{8\pi G}\frac{v_s^2\rho^2}{4r_s^2}\left(\frac{df}{dr_s}\right)^2. \tag{6}$$

2 The ESAA spacetime

One can choose to describe the function of the warp drive in relationship to the function $g(\rho)$, where $\rho = r_s$ and $g(\rho) = 1 - f(\rho)$. We approach this aspect in a manner not to dissimilar to the one chosen by Hiscock [6] where we impose a pseudo metric transformation through the following functions

$$g_{00} = A(\rho)^2 - [v_s(r)\; g(\rho)]^2 \tag{7}$$
$$g_{01} = g_{10} = -v_s\; g(\rho) \tag{8}$$
$$g_{11} = g_{22} = g_{33} = -1 \tag{9}$$

In order to reduce the energy requirements arbitrarily we consider the following lapse function $A(ct, \rho)$. Where have set $A = 1$ both at the location of the ship, and far from it, but allow it to become large in the warped region following the lapse function $A(ct, \rho)$. Such that the above functions (7-9) have the following metric signature in cylindrical coordinates:

$$ds^2 = [(A) - v_s(r)\; g(\rho)]^2 + 2v_s g(\rho) dctdz' - dz'^2 - dr^2 - r^2 d\phi^2. \tag{10}$$

We further define (10) with the following coordinate transformation:

$$z' = z - \int^{ct} v_s dct \tag{11}$$

such that the ship velocity has the following definition $v_s = 1 = c$ thereby arriving at

$$\begin{aligned} ds^2 = &\quad (A(ct,\rho)^2 - v_s(ct)^2 g(\rho)^2) dct^2 - 2v_s(ct) g(\rho)\cos(\theta) dctdr \\ &+ \quad 2v_s(ct) g(\rho)\sin(\theta)\rho\; dctd\theta - dr^2 - \rho^2 d\theta^2 - \rho^2\sin^2(\theta) d\phi^2. \end{aligned} \tag{12}$$

Thus the exact the solution for the Einstein tensor G^{00} for (12) is given by

$$\begin{aligned} G^{ct\;ct} = &\quad -\tfrac{1}{4} v_s(ct)^2 \left(\tfrac{\partial}{\partial\rho}\, g(\rho)\right) \sin^2(\theta) / \big(A(ct,\rho)^2 - v_s(ct)^2 g(\rho)^2 \\ &+ \qquad v_s(ct)^2 g(\rho)^2\cos^2(\theta) + v_s(ct)^2 g(\rho)^2 \sin^2(\theta)\big)^2 \end{aligned} \tag{13}$$

where we have imposed the trigonometric identity $sin^2(\theta) + cos^2(\theta) = 1$.

We now just briefly discuss how the dramatic energy reductions can occur within the warp drive spacetime. First beginning with the Alcubierre metric in the ship frame coordinates which is stated so that the function of interest is $g(\rho)$ instead of $f(\rho)$. Alcubierre originally introduced a gravitational time dilation term that doesn't limit ship speed in the g_{00} term of his metric that has been overlooked thus far. Instead of inserting this term starting with the remote frame metric as Alcubierre did, we have inserted it into the ship frame metric. With these coordinates an exact calculation of the entire stress energy tensor can be made fairly easy. The T^{00} term is the ship frame energy density term debated to be to large. Its solution with the lapse function $A(ct, \rho)$ inserted is

$$T^{00} = -\frac{v_s}{4}\frac{c^4}{8\pi G}\left(\frac{dg}{d\rho}\right)^2 \frac{\{\sin(\theta)\}^2}{\{(A(ct,\rho)\}^4} \tag{14}$$

see section 4 to see how this equation was derived. Letting A become large arbitrarily reduces this negative ship frame energy density requirement. Due to the non vanishing presence of the other stress-energy tensor terms there will still be negative energy in other frames, and thus a Weak Energy Condition (WEC) violation, but we really don't know that the "quantum implied" weak energy condition is a frame covariant principle even though it has been expressed in a frame covariant manner. [$T_{ab}U^aU^b \geq 0$ for all time like U^a.] Since the ship forms the warp, satisfying that there is very little ship frame negative energy may be enough.

3 Quantum Inequalities

Pfenning applied [7] a quantum inequality for a free, massless scalar field to the Alcubierre warp even though the Alcubierre spacetime is not the result of a free, massless scalar field. Because of this, the results for the restriction on the warp shell's thickness are unreliable. It is this quantum inequalities restriction on the warp shell's thickness that causes the negative energy's magnitude to be so great. Therefore this result is also unreliable. Pfenning also did not include the possibility of an $A(ct, \rho)$ term other than $A = 1$. Even though the quantum inequality may not be reliably applicable to the warp drive spacetime we can go back and redo the calculation with a variable $A(ct, \rho)$ included. The quantum inequality is

$$\frac{\tau_0}{\pi}\int_{-\infty}^{+\infty} \frac{\langle T^{\mu\nu}U_\mu U_\nu\rangle}{\tau^2+\tau_0^2} d\tau \geq \frac{h}{c}\frac{3}{32\pi^2\tau_0^4} \tag{15}$$

In order to apply it we must find $T^{\mu\nu}U^\mu U^\nu$ and τ_0 for a Eulerian observer. A Eulerian observer is an observer who starts out just inside the warp shell at its equator with zero initial velocity with a *small sampling time* according to the ship frame who "samples" the time it takes τ_0 for the negative energy region to pass him.

From the above interval we have eq. (10)

$$c^2 = c^2(A^2 - v_S^2 g^2)(dt/d\tau)^2 - 2v_s gc(dt/d\tau)U - U^2 \tag{16}$$

thus from the definition of the lapse function we have

$$\frac{dt}{d\tau} = A^{-1}. \tag{17}$$

Combining these (with a minimum sampling time) results in

$$0 = \frac{1}{2}U^2 + v_s gcA^{-1}U + \frac{1}{2}v_s^2 g^2 A^{-2} \tag{18}$$

$$U = -v_s g c A^{-1} \tag{19}$$

$$[U^\mu] = \begin{bmatrix} cA^{-1} \\ -v_s g c A^{-1} \\ 0 \\ 0 \end{bmatrix} \tag{20}$$

$$[U_\mu] = [\ cA \quad 0 \quad 0 \quad 0\ ]. \tag{21}$$

By inserting $T^{\mu\nu}U_\mu U_\nu = A^2 T^{00} c^2$ into the inequality (15) we have

$$\frac{\tau_0}{\pi}\int_{-\infty}^{+\infty} \frac{A^2 T^{00}}{\tau^2+\tau_0^2} d\tau \geq \frac{h}{c^3}\frac{3}{32\pi^2\tau_0^4} \tag{22}$$

and

$$\tau_0 \int_{-\infty}^{+\infty} \frac{\beta^2}{A^2\rho^2}\left(\frac{dg}{d\rho}\right)^2 \frac{d\tau}{\tau^2+\tau_0^2} \leq \frac{Gh}{c^7}\frac{3}{r^2\tau_0^4} \tag{23}$$

where we can make the replacement

$$\rho^2 = r^2 + z^2 \tag{24}$$

with $r = const.$

Pfenning then makes an approximation of the geodesic motion of the Eulerian observer. Equation eq. 5.11 (note further references to Pfenning in this section refer to [7]). Since we are using the ship frame with $A(ct,\rho)$ inserted, we must include its effect here as well

$$z \approx -vg(r)A^{-1}(r)\tau \tag{25}$$

$$\rho^2 = r^2 + v^2 g^2 A^{-2}\tau^2 \tag{26}$$

such that

$$\tau_0 \int_{-\infty}^{+\infty} \frac{v^2}{A^2 r^2 + v^2 g^2 \tau^2}\left(\frac{dg}{d\rho}\right)^2 \frac{d\tau}{\tau^2+\tau_0^2} \leq \frac{G\hbar}{c^5}\frac{3}{r^2\tau_0^4}. \tag{27}$$

We make a definition corresponding to Pfenning's equation 5.15

$$\psi = \frac{r}{[vg(r)]} \tag{28}$$

And according to Pefenning 5.16 the integral should be approximately

$$\frac{\pi}{A\psi(\tau_0 + A\psi)} \leq \frac{G\hbar}{c^5}\frac{3\Delta^2}{v^2\psi^2\tau_0^4} \tag{29}$$

So the inequality becomes

$$\frac{\pi}{3} \leq \frac{G\hbar}{c^5}\frac{\Delta^2[A(r)]^2}{v^2\tau_0^4}\left(\frac{v\tau_0}{rA(r)}g(r) + 1\right). \tag{30}$$

This result is the same as Pfenning eq 5.17 with the exception that $A(r)$ is not necessarily 1. Pfenning then asserts that this result must hold for sample times that are small compared to the square root inverse of the largest magnitude of the Riemann tensor components as calculated for the local observers frame.

If for simplicity A is kept approximately constant through the negative energy region, then the largest Riemann tensor component for this spacetime is

$$\left|R^{\rho}_{00\rho}\right| \approx \beta^2 (dg/d\rho)^2 \ (v/\Delta)^2 \tag{31}$$

So in this case, the sampling time must be restricted to

$$\tau_0 = \alpha\Delta/v \tag{32}$$

where $0 < \alpha < 1$.

Inserting this into the inequality and looking at the case of large A approximately constant A_0 through the negative energy region leads to

$$\Delta \leq (3/\pi)^{1/2}(Gh/2\pi c^3)^{1/2}(v/c)A_0/\alpha^2 \tag{33}$$

Choosing $\alpha = 0.10$, this approximates to

$$\Delta \leq 10^2 (v/c) l_{Pl} A_0 \tag{34}$$

Now we see that letting A become arbitrarily large also arbitrarily thickens the minimum warp shell thickness. Therefore the -0.068 solar mass calculation [7] had a reachable shell thickness. All that remains then is to divide the Pfenning -0.068 solar mass result by an A_0^4 which would allow the thickness chosen, and which will lower the energy magnitude even farther by several orders of magnitude.

4 Total Energy Calculations

The energy requirements for the warp drive spacetime can be calculated from

$$E = \int T^{00} dV \tag{35}$$

For T^{00} we have

$$T^{00} = -\frac{v_s c^4}{32\pi G}\left(\frac{dg}{d\rho}\right)^2 \frac{r^2}{r^2+z^2}\frac{1}{A^4} \tag{36}$$

Where we assume $A = const$ to be large, one can also see that this is identical to eq. (14). We now choose

$$dV = \rho^2 \sin(\theta) d\rho d\theta d\phi$$

so that we can write

$$E = -v_s^2 \frac{c^4}{12G}\int_0^\infty \rho^2 \left(\frac{dg}{d\rho}\right)^2 \left(\frac{1}{A^4}\right) d\rho \tag{37}$$

which reduces to

$$E_{ESAA} = -\int_0^\infty \frac{v_s}{12}\frac{c^4}{G}\left(\frac{dg}{d\rho}\right)^2 \left(\frac{1}{A^4}\right)\rho^2 d\rho. \tag{38}$$

Thus from section 3 we can show similarly that the energy can be given from:

$$E = -\int_0^\infty \frac{v_s^2 c^4}{12G}\left(\frac{1}{\Delta}\right)^2 \left(\frac{1}{A^4}\right)\rho^2 d\rho. \tag{39}$$

5 Post relativistic warp drives?

One of the requirements for a warp drive spacetime which has a velocity greater than that of light is the existence of negative energy [2]. However this negative energy, exotic matter, quintessence, or however you chose to define it can be made to pseudoly appear from negative brane tension [8]. More specifically for our case, an Anti de Sitter (AdS) spacetime would b e a logical choice for such an exploration. The reason for this choice is the fact that Λ acts as a scalar field to exploit negative energy densities [4]. In reference to [9] we can write an arbitrary metric for a single "warped" extra dimension that has 3-D rotational invariance in AdS space by

$$ds^2 = -a^2(r,t)dt^2 + b^2(r,t)d\vec{x}^2 + c^2(r,t)dr^2 \tag{40}$$

This equation at a first approximation is identical to eq. (10). Now considering eq. (12) in reference to eq. (40) a five-dimensional AdS warp drive spacetime can be given by

$$\begin{aligned} d\hat{s}^2 = \quad & -\Lambda_{b\kappa} g(\hat{\rho})^2 dt^2 + A(c\hat{t},\hat{\rho})d\Sigma_\kappa^2 + (A(c\hat{t},\hat{\rho}) - v_s(c\hat{t},\hat{\rho})^2 g(\hat{\rho})^2) \\ & dc\hat{t}^2 + d\hat{\rho}^2 + \hat{\rho}^2 d\theta^2 + \hat{\rho}\sin^2\theta d\phi^2 \end{aligned} \tag{41}$$

where

$$d\Sigma_\kappa^2 = \frac{d\sigma^2}{(1-\kappa L^{-2}\sigma^2)} + \sigma^2 d\Omega_2^2 \tag{42}$$

with κ being the curvature scale and L being the length scale of the bulk $\Lambda_{b\kappa}$. Where $ct, \rho = const$ and $d\Sigma$ is a unit metric for a 3 brane. From this the Alcubierre warp drive function $f(\rho)$ expands and contracts within the (3+1) slice of the AdS spacetime via a scale factor dependent on time $(c\hat{t}, 0)$. Since the five-dimensional AdS spacetime contains slices of the four-dimensional space the time coordinates rescale differently at different points in the extra dimension of the de Sitter space. Thus causing the (ct, ρ) terms to scale differently in the 5-D space than it does in the 4-D space, therefore a gravitational wave in 4-D space can appear to exceed the light because the bulk spacetime acquires different velocities for $c = 1$ when moving through the 'extra' dimension. This spacetime would also act to suppress the "need one to make one" paradox of [11]. When the 5-D space is static and there is no gauge field one can have a more general solution

$$ds^2 = -A(ct,\rho)dt^2 + L^{-2}\rho^2 d\Sigma_\kappa^2 + (A(ct,\rho) - v_s(ct,\rho)^2 g(\rho)^2)d\rho^2 \tag{43}$$

with

$$L^{-2} = -\frac{1}{6}\kappa_5^2 \Lambda_{bk} \tag{44}$$

which would essentially describe a warp drive in which $v_0 < c$. What is interesting to note about eq. (41) is that it can make the 4-D warp drive function $g(\rho)$, appear to be generated from a cosmologic term of order $\Lambda g(\rho)$ in AdS space. Meaning that matter within the warp bubble wouldn't need to become tachyonic to solve the horizon [control] problem of the Alcubierre Warp Drive as $c \leq v_0$ (i.e. the scalar field acts to create a hyper warp drive without large negative energy requirements), it is however noted that inducing the fifth dimension is purely a mathematical trick which may have problems of its own. In the tradition of Alcubierre the warp drive of eq. (41) just begs to be named after its familiar counterpart, the "hyperdrive" of science fiction. We also note that a similar but more conventional approach was taken by González-Diáz [10], in which a Alcubierre-Hiscock spacetime is embedded in a three-dimensional Misner space, which fits into the frame work of classical GR as opposed to this discussion.

5.1 a Broeck extended AdS spacetime?

Equation (41), shouldn't look to surprising at first hand, let us look at the Broeck spacetime [12]:

$$ds^2 = -dt^2 + B^2(\rho)[dx - v_s(t)f(\rho)dt)^2 + dy^2 + dz^2] \tag{45}$$

We see that $L^{-2}\rho^2 \equiv B^2(\rho)$, thus (43) can be interpreted as an AdS Broeck spacetime, this is because $B^2(\rho)$ would naturally arise from primed coordinate transformations [13], eq.(11) is a generic example of this. Thus when comparing equation 5 of [12] to (43), something quite odd occurs to the lapse function $A(ct,\rho)$ when $v_0 > c$ effectively increasing the apparent amount of negative energy within the local brane. Moreover a similar scenario for (43) implies a coefficient of order $A^2{\cdot}\Lambda_{bk}g(\hat{\rho})^2$. The resulting total energy of the bulk spacetime in comparison to (38) when $v_0 \geq c$ is thus

$$E_{bk} = \int_0^\infty \frac{v_s c^4}{12G}\frac{1}{A(c\hat{t},\rho)^2}\ \rho^2 d\rho^3 \tag{46}$$

Notice that the bulk comsomologic term Λ_{bk} acts to flip the negative energy our local brane. This can be expected because a brane with negative tension can contribute and act positively within a higher dimensional brane [8], and therefore appear to violate the NEC (Null Energy Condition). This also suggest that $\rho \equiv k_{\mu\nu} = K^+_{\mu\nu} - K^-_{\mu\nu}$, i.e. the center of the Broeck bubble contains a gravitational kink.

We now want to discuss a specific boundary condition to illustrate the properties of a *hyperdrive*. To begin this discussion the lapse function will be given as

$$A(ct,\rho) = \alpha_{PWL} = f_{PWL} \tag{47}$$

thus to arrive at specific boundary condition we replace (4) with a function that resembles eq. 4 of [3] therefore we have the following piecewise function:

$$f_{PWL}(\rho) = \begin{cases} 1 & \rho < R - \frac{\Delta}{2} \\ (1 + [(\rho - R - \frac{\Delta}{2})(\rho - R + \frac{\Delta}{2})]/D)^N & R - \frac{\Delta}{2} < \rho < R + \frac{\Delta}{2} \\ 1 & \rho > R + \frac{\Delta}{2} \end{cases} \tag{48}$$

It is also noted that the $R\pm(\Delta/2)$ term could be a boundary equal to or greater than one corresponding to some value $\alpha \geq 1$ in standard units) and that this value was chosen arbitrarily, however for this instance we have defined it in manner which coincides with Pfenning's analysis throughout this work. N is the warp parameter which describes the magnitude of the warp factor:

$$N(\rho)_{warp} = B(\rho) = 1 + \alpha \tag{49}$$

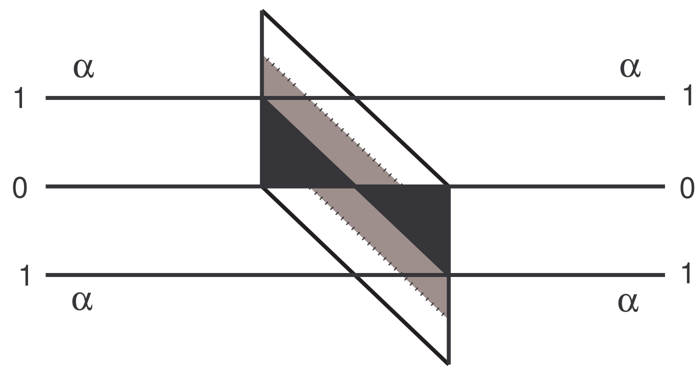

Figure 1: The ESAA spacetime represented one dimensionally in Cartesian coordinates, with $\alpha_{PWL} = 1$ this spacetime appears hyper-dimensional in comparison to the *Alcubierre spacetime* $f(\rho) = 0$. This figure represents the varying warp parameter N the minimum value $N = B(\rho) = 1$ gives the Alcubierre top hat $f(\rho) = 1$ (black). The median (gray) and maximum (white) represents an energy reduction due to a decreased volume as a factor of the warped region. For an arbitrary boundary condition $\alpha_{PWL} = (1 + [(\rho - R - \frac{\Delta}{2})(\rho - R + \frac{\Delta}{2})]/D)^N$, when $R = 100\ m$ the "maximum warp" factor is $N = 9.3$, where the lapse function becomes singular.

this definition describes how the Alcubierre spacetime differs from our own[3].

With the boundary condition (48) the energy requirements for this warp drive spacetime can be given from:

$$\begin{aligned} T^{\alpha\beta} n_\alpha n_\beta &= \alpha^2_{PWL}\, T^{00} \\ &= -(R + \tfrac{\Delta}{2})^{2N}\, G^{00} \\ &= -\left[(R + \tfrac{\Delta}{2})^{2N}\right] \left[v_s^2/4\right] \left[sin^2\theta\right] \left[dg(\rho)/d\rho\right]^2 \end{aligned} \tag{50}$$

With $N = 1$ eq. (46) becomes

$$E_{bk} = \frac{v_s c^4}{144G} \frac{1}{(\hat{R} + \frac{1}{2}\hat{\Delta})^{2N}} \int_0^\infty \rho^4 d\rho \tag{51}$$

or

$$\frac{E_{bk}}{c^2} \approx 1.7 \times 10^{38} g = 1.2 \times 10^5 \cdot M_{Sun}. \tag{52}$$

The ESAA total energy E_{ESAA} is given with $N = 1$ since $v_0 = c$, this depicts an *Alcubierre like* spacetime from footnote 3 we see that

$$N = \int_1^\infty \lim_{\alpha \to 0} \alpha + 1 = 0 + 1 = 1.$$

However, we now have $v_0 > c$, thus $\alpha \geq 1$ in standard units, by comparing eq. (38) to eq. (51) we see that $\alpha = 1$, therefore meaning the hyperdrive is defined

[3]Where $B(\rho)$ is derived from eq. 5 of [12] and the warp parameter N we can also define alpha as $\alpha = N - 1$.

through the following parameter

$$N = \int_1^\infty \alpha + 1 = 1 + 1 = 2.$$

Thus when applying an arbitrary boundary condition $\alpha_{PWL} = (b)^N$ for the warped region, we see that N, would reduce the energy requirements for an arbitrary total energy calculation such as (38). Thus it is seen that under these conditions the velocity of the warp bubble is to be implicitly interpreted as $v_0 = 10^2 c$.

In other words using a Pfenning inspired boundary condition it takes just under one earth mass to achieve a velocity 100 times that of light. From the ship's frame the apparent velocity is much less from [13]:

$$\frac{dx'}{dt'} = \frac{dx/dt\beta}{1 - \beta dx/dt} \tag{53}$$

we see that a fifteen day trip to alpha centauri, would give a local apparent velocity of $v = 0.01002c$, thus compounding the aberration effects of [14].

5.2 metric patching

From this it can be seen why the warp drive violates the standard energy conditions in GR, assuming a negative brane tension one has [8]:

$$T^{\mu\nu} k_\mu k_\nu = -\Lambda_D g^{\mu\nu}_{induced} k_\mu k_\nu \delta^{n-p}(n^a) = \Lambda_D \left[\sum_{a=1}^{n-p} (n^\mu_a k_\mu)^2 \right] \delta^{n-p}(n^a) \tag{54}$$

from this the NEC becomes $T^{\mu\nu} k_\mu k_\nu < 0$. When a killing vector K^μ is inserted in (54), for the right hand side we have

$$T^{\mu\nu} k_\mu k_\nu K^\mu = \alpha T^{\mu\nu} = \Lambda_D \left[\sum_{a=1}^{n-p} (n^\mu_a)^2 \right] \delta^{n-p}(n^a) \tag{55}$$

which is exactly what we would expect from (41), with the presence of a gauge field. In terms of the AdS geometry this is caused by the scale factor $(c\hat{t}, \rho)$, such that the time lapse function is given by the relation $A^2 \Lambda_{bk} = A(c\hat{t}, \rho)^2$, i.e. converting the brane negative energy into positive energy in the bulk frame, as suggested above.

The total energy in the bulk is positive with $r_s \in [\infty, a(t)]$ thus with metric patching the energy in our three brane can be calculated from:

$$G^{\mu\nu} = \eta^{\mu\nu} + \hbar^{\mu\nu} + K^{\mu\nu}_- - K^{\mu\nu}_+ \tag{56}$$

Throughout our analysis of the *hyperdrive* we have considered a metric patching of (3+1) within an embedding in E^4, while in previous works the warp drive has been considered in the form (2+1) embedded in E^3 [10]. A similar construction has also been considered for transversible wormholes [8] in reference to string theory, which conforms to an (n+1) bulk and an ([n-1]+1) brane, again as suggest by Everett the warp drive appears as a special case wormhole (with the exception that this wormhole appears to be a shortcut through *hyperspace*).

6 Summary

The goal we set out was to reduce the energy requirements of the warp drive spacetime to more physically realistic amounts. Our analysis has shown that simply considering a warp drive spacetime with an arbitrary lapse function can dramatically lower the energy requirements within the Einstein tensor. We have shown that Quantum Inequalities are a poor choice for 'curved spacetimes,' but even so minor modifications of QI's can also reduce their allowable energy restrictions. Finally we have derived a mathematical trick which may allow the warp drive spacetime to survive within the superluminal range. In closing it can be shown that the warp drive spacetime can not be ruled out because of 'unphysical energy conditions' alone, by simply choosing an arbitrary function $A(ct, \rho)$ the energy reductions become rather dramatic. It is however noted that such an arbitrary function is merely a mathematical devise, within a four-dimensional spacetime there is no mechanism to set $N > 1$ as far as we know. However, it is somewhat easy to consider $N = 2$ for a possible "hyperdrive" spacetime, with our discussion this would correspond to a five-dimensional AdS spacetime (in which a "warp bubble" could be interpreted to move with a velocity that is 100 times that of light), and without experimental verification of the fifth coordinate this may also be considered a clever trick of its own. It is therefore noted that the warp drive spacetime appears to have several limitations within semi-classical GR, e.g. the violation of the Null Energy Condition (NEC), therefore it is likely that a more plausible warp drive may reside within post relativistic corrections of GR, indeed *Ex Somnium Ad Astra*.

Acknowledgement

The authors would like to express their sincere gratitude towards Miguel Alcubierre for his helpful discussions and appreciated comments throughout the developments of this work.

A Selected exact contrivariant solutions of the Einstein tensor

The exact solutions to the ESAA metric (12) were produced in Maple VI with the aid of GRTensorII and are as follows (and for that reason v_s is represented by β in the following calculations):

$$\begin{aligned} G^{ct\,\rho} = \quad & \tfrac{1}{4}\Big(\big(4A(ct,\rho)^2 + 4\beta(ct)2g(\rho)^2\cos^2(\theta) - 4\beta(ct)^2 g(\rho)^2 \\ + \quad & \beta(ct)^2 g(\rho)\sin^2(\theta)\big)\left(\tfrac{\partial}{\partial\rho}g(\rho)\right)\rho \\ + \quad & 4\beta(ct)^2 g(\rho)\sin^2(\theta)\left(\tfrac{\partial}{\partial\rho}g(\rho)\right)\cos(\theta)\beta(ct)\Big) \\ / \quad & \rho\left(A(ct,\rho)^2 - \beta(ct)^2 g(\rho)^2\cos^2(\theta) + \beta(ct)^2 g(\rho)^2\sin^2(\theta)\right)^2 \end{aligned} \tag{57}$$

$$G^{\rho\,\theta} = \tfrac{1}{4}\Big(\big(2A(ct,\rho)^2\rho\left(\tfrac{\partial}{\partial ct}\beta(ct)\right) + 4A(ct,\rho)^2\beta(ct)^2 g(\rho)\cos(\theta)$$

$$
\begin{aligned}
- &\quad 2\beta(ct)\rho\left(\tfrac{\partial}{\partial ct}A(ct,\rho)\right)A(ct,\rho)+\beta(ct)^4 g(\rho)^2 \\
&\quad \cos(\theta)\sin^2(\theta)\left(\tfrac{\partial}{\partial\rho}g(\rho)\right)\rho \\
+ &\quad 4\beta(ct)^4 g(\rho)^3\cos(\theta)^3-4\beta(ct)^4 g(\rho)^3 \\
&\quad \cos(\theta)+4\beta(ct)^4 g(\rho)^3\cos(\theta)\sin^2(\theta)) \\
&\quad \left(\tfrac{\partial}{\partial\rho}g(\rho)\right)\sin(\theta)\Big)\big(\rho^2\big(A(ct,rho)^2-\beta(ct)^2 g(\rho)^2 \\
+ &\quad \beta(ct)^2 g(\rho)^2\cos^2)(\theta)+\beta(ct)^2 g(\rho)^2\sin^2(\theta)\big)^2\Big)
\end{aligned}
\tag{58}
$$

B Selected exact covariant solutions of the Einstein tensor

Again these are the solutions to the metric (12), with the v_s to β modifications:

$$
\begin{aligned}
G_{\rho\rho} = &\quad \tfrac{1}{4}\big(8A(ct,\rho)\left(\tfrac{\partial}{\partial\rho}A(ct,\rho)\right)+4\beta(ct)^2 g(\rho)\sin^2(\theta)\left(\tfrac{\partial}{\partial\rho}g(\rho)\right)^2\sin^2(\theta)\rho \\
- &\quad 8\beta(ct)^2 g(\rho)\left(\tfrac{\partial}{\partial\rho}g(\rho)\right)\big)\rho\big(A(ct,\rho)^2-\beta(ct)^2 g(\rho)^2 \\
+ &\quad \beta(ct)^2 g(\rho)^2\cos^2(\theta)+\beta(ct)^2 g(\rho)^2\sin^2(\theta)\big)
\end{aligned}
\tag{59}
$$

$$
\begin{aligned}
G_{ct\ ct} = &\quad -\tfrac{1}{4}\beta(ct)^2\big(-4\rho A(ct,\rho)^3\left(\tfrac{\partial^2}{\partial\rho^2}A(ct,\rho)\right)g(\rho)^2\sin^2(\theta) \\
+ &\quad 4\beta(ct)^2 g(\rho)^3\sin^4(\theta)\rho\left(\tfrac{\partial^2}{\partial\rho^2}g(\rho)\right)A(ct,\rho)^2 \\
+ &\quad 4\beta(ct)^2 g(\rho)^3\cos^2(\theta)\sin^2(\theta)\left(\tfrac{\partial^2}{\partial\rho^2}g(\rho)\right)\rho A(ct,\rho)^2 \\
- &\quad 4\rho A(ct,\rho)^2\beta(ct)^2 g(\rho)^3\left(\tfrac{\partial^2}{\partial\rho^2}g(\rho)\right)\sin^2(\theta) \\
- &\quad 4\rho\beta(ct)^2 g(\rho)^4\sin^2(\theta)A(ct,\rho)\left(\tfrac{\partial^2}{\partial\rho^2}A(ct,\rho)\right)\cos^2(\theta) \\
+ &\quad 4g(\rho)\sin^2(\theta)\rho\left(\tfrac{\partial^2}{\partial\rho^2}g(\rho)\right)A(ct,\rho)^4 \\
+ &\quad 4\beta(ct)^2 g(\rho)^3\cos^2(\theta)\sin^2(\theta)\left(\tfrac{\partial^2}{\partial\rho^2}g(\rho)\right)\rho A(ct,\rho)^2 \\
- &\quad 4\rho A(ct,\rho)\beta(ct)^2 g(\rho)^3\left(\tfrac{\partial}{\partial\rho}g(\rho)\right)\left(\tfrac{\partial}{\partial\rho}A(ct,\rho)\right)\sin^2(\theta) \\
+ &\quad 3\rho\beta(ct)^4 g(\rho)^4\sin^2(\theta)\left(\tfrac{\partial}{\partial\rho}g(\rho)\right)^2\cos^2(\theta) \\
+ &\quad 4\beta(ct)^2 g(\rho)^4\sin^2(\theta)A(ct,\rho)\left(\tfrac{\partial}{\partial\rho}A(ct,\rho)\right) \\
- &\quad 4\beta(ct)^2 g(\rho)^3\sin^2(\theta)A(ct,\rho)^2\left(\tfrac{\partial}{\partial\rho}g(\rho)\right) \\
- &\quad 4\beta(ct)^2 g(\rho)^4\sin^4(\theta)A(ct,\rho)\left(\tfrac{\partial}{\partial\rho}A(ct,\rho)\right) \\
- &\quad 8g(\rho)^4\cos^2(\theta)\beta(ct)^2 A(ct,\rho)\left(\tfrac{\partial}{\partial\rho}A(ct,\rho)\right) \\
- &\quad 8g(\rho)^4\cos^4(\theta)\beta(ct)^2 A(ct,\rho)\left(\tfrac{\partial}{\partial\rho}A(ct,\rho)\right) \\
- &\quad 12g(\rho)^4\cos^2(\theta)\beta(ct)^2 A(ct,\rho)\left(\tfrac{\partial}{\partial\rho}A(ct,\rho)\right)\sin^2(\theta)
\end{aligned}
$$

$$
\begin{array}{lc}
- & 8g(\rho)^3\cos^2(\theta)\beta(ct)^2\left(\frac{\partial}{\partial\rho}g(\rho)\right)A(ct,\rho)^2 \\
- & 3\beta(ct)^2\sin^4(\theta)\left(\frac{\partial}{\partial\rho}g(\rho)\right)^2\rho A(ct,\rho)^2 g(\rho)^2 \\
- & 3\beta(ct)^2\sin^2(\theta)\left(\frac{\partial}{\partial\rho}g(\rho)\right)^2\rho g(\rho)^2\cos^2(\theta)A(ct,\rho)^2 \\
+ & 4\rho\beta(ct)^2 g(\rho)^4\left(\frac{\partial}{\partial\rho}A(ct,\rho)\right)^2\sin^2(\theta) \\
- & 3\rho\beta(ct)^4 g(\rho)^4\left(\frac{\partial}{\partial\rho}g(\rho)\right)^2\sin^2(\theta) \\
- & 4\rho\beta(ct)^2 g(\rho)^4\left(\frac{\partial}{\partial\rho}A(ct,\rho)\right)^2 \\
+ & 3\rho\beta(ct)^4 g(\rho)^4\sin^4(\theta)\left(\frac{\partial}{\partial\rho}g(\rho)\right)^2 \\
+ & 6\rho A(ct,\rho)^2\beta(ct)^2\left(\frac{\partial}{\partial\rho}g(\rho)\right)^2 g(\rho)^2\sin^2(\theta) \\
- & 4\rho g(\rho)\sin^2 A(ct,\rho)^3\left(\frac{\partial}{\partial\rho}A(ct,\rho)\right)\left(\frac{\partial}{\partial\rho}g(\rho)\right) \\
+ & 4\rho\beta(ct)^2 g(\rho)^3\sin^2(\theta)A(ct,\rho)\left(\frac{\partial}{\partial\rho}A(ct,\rho)\right)\left(\frac{\partial}{\partial\rho}g(\rho)\right)\cos^2(\theta) \\
+ & 4\rho\beta(ct)^2 g(\rho)^3\sin^4(\theta)A(ct,\rho)\left(\frac{\partial}{\partial\rho}A(ct,\rho)\right)\left(\frac{\partial}{\partial\rho}g(\rho)\right) \\
- & 4\rho\beta(ct)^2 g(\rho)^4\sin(\theta)^2\left(\frac{\partial}{\partial\rho}A(ct,\rho)\right)^2\cos^2(\theta) \\
+ & 8\beta(ct)^2 g(\rho)^3\cos^4(\theta)\left(\frac{\partial}{\partial\rho}g(\rho)\right)A(ct,\rho)^2 \\
+ & 12\beta(ct)^2 g(\rho)^3\cos^2(\theta)\left(\frac{\partial}{\partial\rho}g(\rho)\right)A(ct,\rho)^2\sin^2(\theta) \\
+ & 4\beta(ct)^2 g(\rho)^3\sin^4(\theta)\left(\frac{\partial}{\partial\rho}g(\rho)\right)A(ct,\rho)^2 \\
- & 8g(\rho)^2\cos^2(\theta)A(ct,\rho)^3\left(\frac{\partial}{\partial\rho}A(ct,\rho)\right) \\
- & 4g(\rho)^2\sin^2 A(ct,\rho)^3\left(\frac{\partial}{\partial\rho}A(ct,\rho)\right) \\
+ & \sin^2(\theta)\left(\frac{\partial}{\partial\rho}g(\rho)\right)^2\rho A(ct,\rho)^4 \\
+ & 8g(\rho)\cos^2(\theta)\left(\frac{\partial}{\partial\rho}g(\rho)\right)A(ct,\rho)^4 \\
+ & 4g(\rho)\sin^2(\theta)\left(\frac{\partial}{\partial\rho}g(\rho)\right)A(ct,\rho)^4) \\
/ & \big(A(ct,\rho)^2-\beta(ct)^2 g(\rho)^2+\beta(ct)^2 g(\rho)^2\cos^2(\theta) \\
+ & \beta(ct)^2 g(\rho)^2\sin^2(\theta)\big)^2\rho
\end{array}
\tag{60}
$$

References

[1] Alcubierre M. The Warp Drive: Hyper-Fast Travel Within General Relativity *Class. Quantum Grav.* **11** (1994), L73–77. gr-qc/0009013

[2] Olum K. Superluminal travel requires negative energies. *Phys. Rev. Lett.* **81** (1998) 3567–70 gr-qc/9805003

[3] Ford L. and Pfenning M. The unphysical nature of “Warp Drive” *Class. Quantum Grav.* **14** (1997) 1743 gr-qc/9702026

[4] Ford L. and Roman T. Classical Scalar Fields and the Generalized Second Law. gr-qc/0009076

[5] Perlmutter S, et al. Measurments of Ω and Λ From 42 High–Redshift Supernovae. *Astrophys. J.* **517** (1999) 565–86 astro-ph/9812133

[6] Hiscock W. Quantum Effects in the Alcubierre warp drive spacetime. *Class. Quantum Grav.* **14** (1997) L183–88. gr-qc/9707024

[7] Pfenning M. Quantum Inequality Restrictions on Negative Energy Densities in Curved Spacetimes. gr-qc/9805037

[8] Barceló C. and Visser M. *Nuc. Phys B* **584** (2000) 415–435

[9] Csáki C, Erlich J, and Grojean C. Gravitational Lorentz Violations and Adjustment of the Cosmological Constant in Asymmetrically Warped Spacetimes. hep-th/0012143

[10] González-Diáz P. *Phys. Rev. D* **62** (2000) 044005

[11] Coule D. *Class. Quantum Grav.* **15** (1998) 2523

[12] Broeck C. A ‘warp drive’ with more reasonable total energy requirements *Class. Quantum Grav.* **16** (1999) 3973–3979 gr-qc/9905084

[13] Everett A. *Phys. Rev. D* **53** (1996) 7365

[14] Clark C, Hiscock W, and Larson S. Null geodesics in the Alcubierre warp drive spacetime: the view from the bridge gr-qc/9907019