

A ‘warp drive’ with more reasonable total energy requirements

Chris Van Den Broeck[†]

Instituut voor Theoretische Fysica,
Katholieke Universiteit Leuven, B-3001 Leuven, Belgium

Abstract

I show how a minor modification of the Alcubierre geometry can dramatically improve the total energy requirements for a ‘warp bubble’ that can be used to transport macroscopic objects. A spacetime is presented for which the total negative mass needed is of the order of a few solar masses, accompanied by a comparable amount of positive energy. This puts the warp drive in the mass scale of large traversable wormholes. The new geometry satisfies the quantum inequality concerning WEC violations and has the same advantages as the original Alcubierre spacetime.

[†] chris.vandenbroeck@fys.kuleuven.ac.be

1 Introduction

In recent years, ways of effective superluminal travel (EST) within general relativity have generated a lot of attention [1, 2, 3, 4, 5]. In the simplest definition of superluminal travel, one has a spacetime with a Lorentzian metric that is Minkowskian except for a localized region S . When using coordinates such that the metric is $\text{diag}(-1, 1, 1, 1)$ in the Minkowskian region, there should be two points (t_1, x_1, y, z) and (t_2, x_2, y, z) located outside S , such that $x_2 - x_1 > t_2 - t_1$, and a causal path connecting the two. This was a definition given in [4]. An example is the Alcubierre spacetime [1] if the warp bubble exists only for a finite time. Note that the definition does not restrict the energy-momentum tensor in S . Such spacetimes will violate at least one of the energy conditions (the weak energy condition or WEC). In the case of the Alcubierre spacetime, the situation is even worse: part of the energy in region S is moving tachyonically [2, 10]. The ‘Krasnikov tube’ [2] was an attempt to improve on the Alcubierre geometry. In this paper, we will stick to the Alcubierre spacetime such as it is. It is not unimaginable that some modification of the geometry will make the problem of tachyonically moving energy go away without changing the other essential features, but we leave that for future work. Here we will concentrate on another problem.

Alcubierre’s idea was to start with flat spacetime, choose an arbitrary curve, and then deform spacetime in the immediate vicinity in such a way that the curve becomes a timelike geodesic, at the same time keeping most of spacetime Minkowskian. A point on the geodesic is surrounded by a ‘bubble’ in space. In the front of the bubble spacetime contracts, in the back it expands, so that whatever is inside is ‘surfing’ through space with a velocity v_s with respect to an observer in the Minkowskian region. The metric is

$$ds^2 = -dt^2 + (dx - v_s(t)f(r_s)dt)^2 + dy^2 + dz^2 \quad (1)$$

for a warp drive moving in the x direction. $f(r_s)$ is a function which for small enough r_s is approximately equal to one, becoming exactly one in $r_s = 0$ (this is the ‘inside’ of the bubble), and goes to zero for large r_s (‘outside’). r_s is given by

$$r_s(t, x, y, z) = \sqrt{(x - x_s(t))^2 + y^2 + z^2}, \quad (2)$$

where $x_s(t)$ is the x coordinate of the central geodesic, which is parametrized by coordinate time t , and $v_s(t) = \frac{dx_s}{dt}(t)$. A test particle in the center of the bubble is not only weightless and travels at arbitrarily large velocity with respect to an observer in the large r_s region, it also does not experience any time dilatation.

Unfortunately, this geometry violates the strong, dominant, and especially the weak energy condition. This is not a problem per se, since situations are known

in which the WEC is violated quantum mechanically, such as the Casimir effect. However, Ford and Roman [6, 7, 8, 9] suggested an uncertainty-type principle which places a bound on the extent to which the WEC is violated by quantum fluctuations of scalar and electromagnetic fields: The larger the violation, the shorter the time it can last for an inertial observer crossing the negative energy region. This so-called quantum inequality (QI) can be used as a test for the viability of would-be spacetimes allowing superluminal travel. By making use of the QI, Ford and Pfenning [3] were able to show that a warp drive with a macroscopically large bubble must contain an unphysically large amount of negative energy. This is because the QI restricts the bubble wall to be very thin, and for a macroscopic bubble the energy is roughly proportional to R^2/Δ , where R is a measure for the bubble radius and Δ for its wall thickness. It was shown that a bubble with a radius of 100 meters would require a total negative energy of at least

$$E \simeq -6.2 \times 10^{62} v_s \text{ kg}, \quad (3)$$

which, for $v_s \simeq 1$, is ten orders of magnitude bigger than the total positive mass of the entire visible Universe. However, the same authors also indicated that warp bubbles are still conceivable if they are microscopically small. We shall exploit this in the following section.

The aim of this paper is to show that a trivial modification of the Alcubierre geometry can have dramatic consequences for the total negative energy as calculated in [3]. In section 2, I will explain the change in general terms. In section 3, I shall pick a specific example and calculate the total negative energy involved. In the last section, some drawbacks of the new geometry are discussed.

Throughout this note, we will use units such that $c = G = \hbar = 1$, except when stated otherwise.

2 A modification of the Alcubierre geometry

We will solve the problem of the large negative energy by keeping the *surface area* of the warp bubble itself microscopically small, while at the same time expanding the spatial *volume* inside the bubble. The most natural way to do this is the following:

$$ds^2 = -dt^2 + B^2(r_s)[(dx - v_s(t)f(r_s)dt)^2 + dy^2 + dz^2]. \quad (4)$$

For simplicity, the velocity v_s will be taken constant. $B(r_s)$ is a twice differentiable function such that, for some \tilde{R} and $\tilde{\Delta}$,

$$B(r_s) = 1 + \alpha \quad \text{for } r_s < \tilde{R},$$

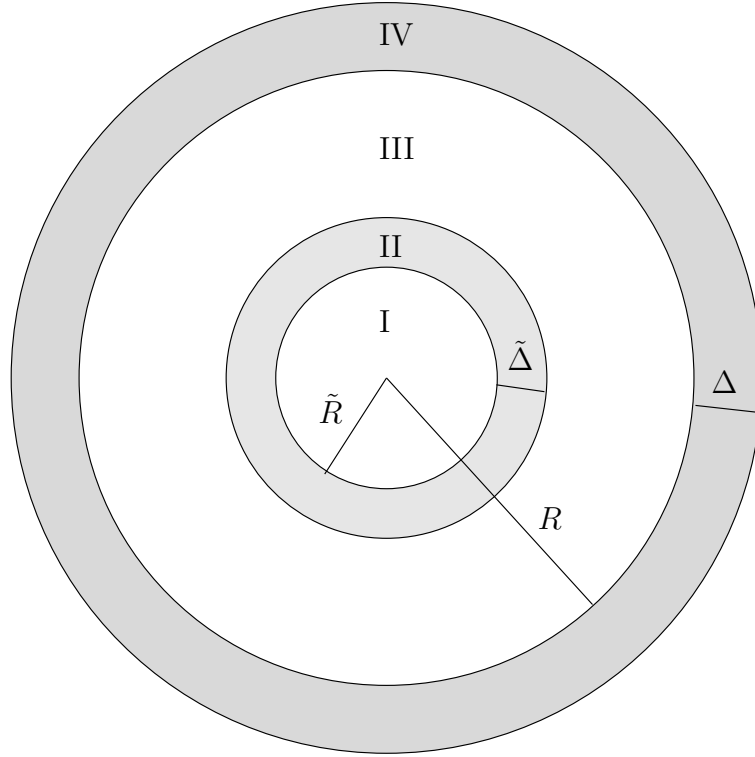


Figure 1: Region I is the ‘pocket’, which has a large inner metric diameter. II is the transition region from the blown-up part of space to the ‘normal’ part. It is the region where B varies. From region III outward we have the original Alcubierre metric. Region IV is the wall of the warp bubble; this is the region where f varies. Spacetime is flat, except in the shaded regions.

$$\begin{aligned}
 1 < B(r_s) \leq 1 + \alpha & \quad \text{for } \tilde{R} \leq r_s < \tilde{R} + \tilde{\Delta}, \\
 B(r_s) = 1 & \quad \text{for } \tilde{R} + \tilde{\Delta} \leq r_s,
 \end{aligned}
 \tag{5}$$

where α will in general be a very large constant; $1 + \alpha$ is the factor by which space is expanded. For f we will choose a function with the properties

$$\begin{aligned}
 f(r_s) &= 1 & \text{for } r_s < R, \\
 0 < f(r_s) &\leq 1 & \text{for } R \leq r_s < R + \Delta, \\
 f(r_s) &= 0 & \text{for } R + \Delta \leq r_s,
 \end{aligned}$$

where $R > \tilde{R} + \tilde{\Delta}$. See figure 1 for a drawing of the regions where f and B vary.

Notice that this metric can still be written in the 3+1 formalism, where the shift vector has components $N^i = (-v_s f(r_s), 0, 0)$, while the lapse function is identically 1.

A spatial slice of the geometry one gets in this way can be easily visualized in the ‘rubber membrane’ picture. A small Alcubierre bubble surrounds a neck leading to a ‘pocket’ with a large internal volume, with a flat region in the middle. It is easily

calculated that the center $r_s = 0$ of the pocket will move on a timelike geodesic with proper time t .

3 Building a warp drive

In using the metric (4), we will build a warp drive with the restriction in mind that all features should have a length larger than the Planck length L_P . One structure at least, the warp bubble wall, cannot be made thicker than approximately one hundred Planck lengths for velocities v_s in the order of 1, as proven in [3]:

$$\Delta \leq 10^2 v_s L_P. \quad (6)$$

We will choose the following numbers for α , $\tilde{\Delta}$, \tilde{R} , and R :

$$\begin{aligned} \alpha &= 10^{17}, \\ \tilde{\Delta} &= 10^{-15} \text{ m}, \\ \tilde{R} &= 10^{-15} \text{ m}, \\ R &= 3 \times 10^{-15} \text{ m}. \end{aligned} \quad (7)$$

The outermost surface of the warp bubble will have an area corresponding to a radius of approximately 3×10^{-15} m, while the inner diameter of the ‘pocket’ is 200 m. For the moment, these numbers will seem arbitrary; the reason for this choice will become clear later on.

Ford and Pfenning [3] already calculated the minimum amount of negative energy associated with the warp bubble:

$$E_{IV} = -\frac{1}{12} v_s^2 \left(\frac{(R + \frac{\Delta}{2})^2}{\Delta} + \frac{\Delta}{12} \right), \quad (8)$$

which in our case is the energy in region IV. The expression is the same (apart from a change due to our different conventions) because $B = 1$ in this region, and the metric is identical to the original Alcubierre metric. For an R as in (7) and taking (6) into account, we get approximately

$$E_{IV} \simeq -6.3 \times 10^{29} v_s \text{ kg}. \quad (9)$$

Now we calculate the energy in region II of the figure. In this region, we can choose an orthonormal frame

$$\begin{aligned} e_{\hat{0}} &= \partial_t + v_s \partial_x, \\ e_{\hat{i}} &= \frac{1}{B} \partial_i \end{aligned} \quad (10)$$

($i = x, y, z$). In this frame, there are geodesics with velocity $u^{\hat{\mu}} = (1, 0, 0, 0)$, called ‘Eulerian observers’ [1]. We let the energy be measured by a collection of these observers who are temporarily swept along with the warp drive. Let us consider the energy density they measure locally in the region II, at time $t = 0$, when $r_s = r = (x^2 + y^2 + z^2)^{1/2}$. It is given by

$$T_{\hat{\mu}\hat{\nu}}u^{\hat{\mu}}u^{\hat{\nu}} = T^{\hat{0}\hat{0}} = \frac{1}{8\pi} \left(\frac{1}{B^4}(\partial_r B)^2 - \frac{2}{B^3}\partial_r\partial_r B - \frac{4}{B^3}\partial_r B \frac{1}{r} \right). \quad (11)$$

We will have to make a choice for the B function. It turns out that the most obvious choices, such as a sine function or a low-order polynomial, lead to pathological geometries, in the sense that they have curvature radii which are much smaller than the Planck length. This is due to the second derivative term, which is also present in the expressions for the Riemann tensor components and which for these functions takes enormous absolute values in a very small region near $r = \tilde{R} + \tilde{\Delta}$. To avoid this, we will choose for B a polynomial which has a vanishing second derivative at $r = \tilde{R} + \tilde{\Delta}$. In addition, we will demand that a large number of derivatives vanish at this point. A choice that meets our requirements is

$$B = \alpha(-(n-1)w^n + nw^{n-1}) + 1, \quad (12)$$

with

$$w = \frac{\tilde{R} + \tilde{\Delta} - r}{\tilde{\Delta}} \quad (13)$$

and n sufficiently large.

As an example, let us choose $n = 80$. Then one can check that $T^{\hat{0}\hat{0}}$ will be negative for $0 \leq w \leq 0.981$ and positive for $w > 0.981$. It has a strong negative peak at $w = 0.349$, where it reaches the value

$$T^{\hat{0}\hat{0}} = -4.9 \times 10^2 \frac{1}{\tilde{\Delta}^2}. \quad (14)$$

We will use the same definition of total energy as in [3]: we integrate over the densities measured by the Eulerian observers as they cross the spatial hypersurface determined by $t = 0$. If we restrict the integral to the part of region II where the energy density is negative, we get

$$\begin{aligned} E_{II,-} &= \int_{II,-} d^3x \sqrt{|g_S|} T_{\hat{\mu}\hat{\nu}}u^{\hat{\mu}}u^{\hat{\nu}} \\ &= 4\pi\tilde{\Delta} \int_0^{0.981} dw (2-w)^2 B(w)^3 \tilde{T}^{\hat{0}\hat{0}}(w) \\ &= -1.4 \times 10^{30} \text{kg} \end{aligned} \quad (15)$$

where $\tilde{T}^{\hat{0}\hat{0}}$ is the energy density with length expressed in units of $\tilde{\Delta}$, and $g_S = B^6$ is the determinant of the spatial metric on the surface $t = 0$. In the last line we have reinstated the factor c^2/G to get the right answer in units of kg. The amount of positive energy in the region $w > 0.981$ is

$$E_{II,+} = 4.9 \times 10^{30} \text{kg}. \quad (16)$$

Both $E_{II,-}$ and $E_{II,+}$ are in the order of a few solar masses. Note that as long as α is large, these energies do not vary much with α if $\tilde{R} = \tilde{\Delta}$ and $\alpha\tilde{R} = 100 \text{m}$. The value of R in (7) is roughly the largest that keeps $|E_{IV}|$ below a solar mass for $v_s \simeq 1$.

We will check whether the QI derived by Ford and Roman is satisfied for the Eulerian observers. The QI was originally derived for flat spacetime [6, 7, 8, 9], where for massless scalar fields it states that

$$\frac{\tau_0}{\pi} \int_{-\infty}^{+\infty} d\tau \frac{\langle T_{\mu\nu} u^\mu u^\nu \rangle}{\tau^2 + \tau_0^2} \geq -\frac{3}{32\pi^2 \tau_0^4} \quad (17)$$

should be satisfied for all inertial observers and for all ‘sampling times’ τ_0 . In [11], it was argued that the inequality should also be valid in curved spacetimes, provided that the sampling time is chosen to be much smaller than the minimum curvature radius, so that the geometry looks approximately flat over a time τ_0 .

The minimum curvature radius is determined by the largest component of the Riemann tensor. It is easiest to calculate this tensor after performing a local coordinate transformation $x' = x - v_s t$ in region II, so that the metric becomes

$$g_{\mu\nu} = \text{diag}(-1, B^2, B^2, B^2). \quad (18)$$

Without loss of generality, we can limit ourselves to points on the line $y = z = 0$; in the coordinate system we are using, the metric is spherically symmetric and has no preferred directions. Transformed to the orthonormal frame (10), the largest component (in absolute value) of the Riemann tensor is

$$R_{\hat{1}\hat{2}\hat{1}\hat{2}} = \frac{1}{B^4} (\partial_r B)^2 - \frac{1}{B^3} \partial_r^2 B - \frac{1}{B^3} \partial_r B \frac{1}{r}. \quad (19)$$

The minimal curvature radius can be calculated using the value of $R_{\hat{1}\hat{2}\hat{1}\hat{2}}$ where its absolute value is largest, namely at $w = 0.348$. This yields

$$\begin{aligned} r_{c,min} &= \frac{1}{\sqrt{|R_{\hat{1}\hat{2}\hat{1}\hat{2}}|}} \\ &= \frac{\tilde{\Delta}}{72.5} \\ &= 1.4 \times 10^{-34} \text{m}, \end{aligned} \quad (20)$$

which is about ten Planck lengths. (Actually, the choice $n = 80$ in (12) was not entirely arbitrary; it is the value that leads to the largest minimum curvature radius.) For the sampling time we choose

$$\tau_0 = \beta r_{c,min}, \quad (21)$$

where we will take $\beta = 0.1$. Because $T^{\hat{0}\hat{0}}$ doesn't vary much over this time, the QI (17) becomes

$$T^{\hat{0}\hat{0}} \geq -\frac{3}{32\pi^2\tau_0^4}. \quad (22)$$

Taking into account the hidden factors c^2/G on the left and \hbar/c on the right, the left hand side is about $-6.6 \times 10^{93} \text{ kg/m}^3$ at its smallest, while the right hand side is approximately $-9.2 \times 10^{94} \text{ kg/m}^3$. We conclude that the QI is amply satisfied.

Thus, we have proven that the total energy requirements for a warp drive need not be as stringent as for the original Alcubierre drive.

4 Final remarks

By only slightly modifying the Alcubierre spacetime, we succeeded in spectacularly reducing the amount of negative energy that is needed, while at the same time retaining all the advantages of the original geometry. The spacetime and the simple calculation I presented should be considered as a proof of principle concerning the total energy required to sustain a warp drive geometry. This doesn't mean that the proposal is realistic. Apart from the fact that the total energies are of stellar magnitude, there are the unreasonably large energy *densities* involved, as was equally the case for the original Alcubierre drive. Even if the quantum inequalities concerning WEC violations are satisfied, there remains the question of generating enough negative energy. Also, the geometry still has structure with sizes only a few orders of magnitude above the Planck scale; this seems to be generic for spacetimes allowing superluminal travel.

However, what was shown is that the energies needed to sustain a warp bubble are much smaller than suggested in [3]. This means that a modified warp drive roughly falls in the mass bracket of a large traversable wormhole [12]. However, the warp drive has trivial topology, which makes it an interesting spacetime to study.

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