Traversable Lorentzian Wormholes: An Overview

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Abstract

An overview of the history and relevance of wormholes is given. Morris and Thorne's description of a traversable wormhole is revisited, and the requirement for exotic matter is derived. The null, weak, strong, dominant, averaged null, averaged weak, and averaged strong energy conditions and the ways that traversable wormholes violate them is discussed. The connection between topological censorship and the averaged null energy condition is shown, with the result that wormholes are not topologically censored. The existence of a method for extracting a sufficient amount of exotic matter from the vacuum stress energy to build a wormhole with an arbitrarily large throat is proven. The connection between wormholes and time machines is explored, and how theorems related to closed timelike curves (chronology protection) or global hyperbolicity (cosmic censorship) might prevent their creation. Finally, it is stated how physical theories might be reconciled to the notion of traversable Lorentzian wormholes.

1. Introduction and Overview

Wormholes have excited interest amongst the general public and researchers alike. For the general public, they provide excellent fodder for imagination by allowing for faster than light travel, time machines, and gateways to other universes. For academia, they provide interesting tests for general relativity, cosmology, causality, Kaluza-Klein theory, brane-worlds, superstring theory, and quantum gravity.

A large body of research exists on the topic of wormholes. Einstein and Rosen first discussed a "bridge" connecting two "sheets" in their 1935 paper[1]; their interest was in describing electrical particles. Wheeler discussed "geons", or self-gravitating bundles of electromagnetic fields in his 1955 paper[2], and gave the first diagram of a "doubly-connected" space. Wheeler subsequently coined the term "wormhole", although his wormholes were at the quantum scale. The seminal work of Morris and Thorne[3] identified the main concepts behind traversable wormholes, and Morris, Thorne, and Yurtsever[4] further developed the energy condition requirements for wormholes and their conversion into time machines. Subsequently, Visser wrote a technical monograph[5] which thoroughly surveyed the research landscape as of 1995. After Visser's work, a large number of papers have been written which clarify, support, or contradict much of the previous research.

The organization of this paper is as follows: 1) A review of the theoretical constraints on the creation and existence of traversable, Lorentzian wormholes (TLWs). 2) A discussion of the energy conditions required for TLWs and their practicality with current known theories 3) Engineering considerations 4) Use of TLWs to create time machines and the implications for their existence 5) Conclusions.

2. Traversable Lorentzian Wormholes

"A wormhole is any compact region of spacetime with a topologically simple boundary but a topologically non-trivial interior." [Visser, p. 89]

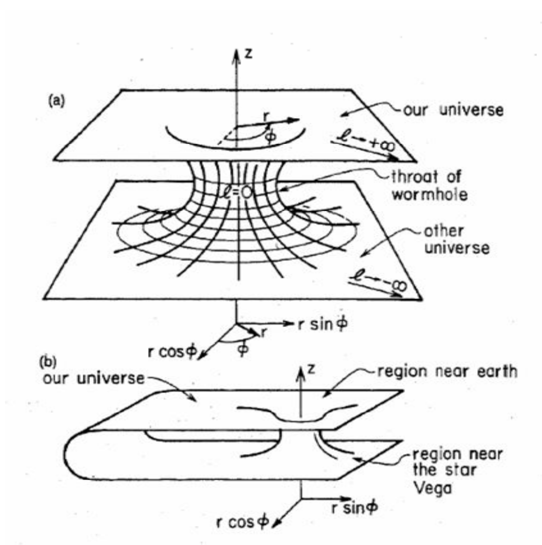

Fig. 1. from MT, adapted from MTW

An idea of the issues can be gleaned by first considering non-traversable wormholes. Einstein-Rosen bridges and Kerr wormholes are really black holes containing event horizons, which forbid egress and kill would-be travelers [Visser, pp. 46-47 and MTW[6], §31]. Schwarzschild wormholes possess extreme tidal forces (unless very large), have throats that collapse too quickly for even light to traverse, and are unstable against small perturbations [MT, §I.B]. Wheeler wormholes are Planck-scale sized [Visser, p. 100]. Euclidean wormholes are probably unphysical since the strong equivalence principle, which states that spacetime is everywhere a Lorentzian manifold, seems to hold [Visser, p. 67].

Morris and Thorne's 1988 paper considered traversable wormholes of the following type:

1. Spherically symmetric, static metric
2. Solutions of the Einstein field equations
3. Throat connecting two asymptotically flat regions of spacetime
4. No event horizons
5. Bearable tidal gravitational forces as experienced by travelers
6. Reasonable transit times with respect to all observers
7. "Physically reasonable" stress energy tensor
8. Stable against perturbations
9. "Physically reasonable" construction materials

Later work relaxes or removes some of these conditions, as we shall see. The following arguments parallel the discussion in MT, §III, and Visser, §11.2.

The metric for a TLW satisfying these conditions is given by [MT, Eq. (1) in natural units]:

$$ds^2 = -e^{2\Phi} dt^2 + \frac{1}{1-b/r} dr^2 + r^2[d\theta^2 + \sin^2\theta d\varphi^2] \tag{1}$$

Φ determines gravitational redshift and is called the redshift function; b determines the spatial shape and is called the shape function. The radial coordinate r, which gives the circumference of the wormhole, behaves somewhat strangely. It decreases from +∞ to a minimum value, b_0, on the lower sheet, and then increases to +∞ on the upper sheet. This minimum value is called the throat of the wormhole, and the proper radial distance is given by [MT, Eq. (32)]:

$$l(r) = \pm\int_{r_0}^{r} \frac{dr'}{\sqrt{1-b(r)/r}} \tag{2}$$

At the wormhole throat, dr/dl =0 since the throat is the minimum value of r. This implies that dl/dr→∞, so that b (r_0) =b_0. In other words, the radius of the wormhole throat (given by r) is b_0 at proper radial distance 0 (given by l).

Using orthonormal basis vectors [MT, Eq. (6)]:

$$
\begin{aligned}
\mathbf{e}_{\hat{t}} &= e^{-\Phi}\mathbf{e}_t \\
\mathbf{e}_{\hat{r}} &= (1-\frac{b}{r})\mathbf{e}_r \\
\mathbf{e}_{\hat{\theta}} &= \frac{1}{r}\mathbf{e}_\theta \\
\mathbf{e}_{\hat{\varphi}} &= \frac{1}{r\sin\theta}\mathbf{e}_\varphi
\end{aligned}
\tag{3}
$$

The nonzero components of the Einstein tensor are [Visser, Eqs. 11.26-28]:

$$
\begin{aligned}
G_{\hat{t}\hat{t}} &= \frac{b'}{r^2} \\
G_{\hat{r}\hat{r}} &= -\frac{b}{r^3} + 2(1-\frac{b}{r})\frac{\Phi'}{r} \\
G_{\hat{\theta}\hat{\theta}} = G_{\hat{\varphi}\hat{\varphi}} &= \left(1-\frac{b}{r}\right)\left[\Phi'' + \Phi'\left(\Phi' + \frac{1}{r}\right)\right] - \frac{1}{2r^2}\left[b'r - b\right]\left(\Phi' + \frac{1}{r}\right)
\end{aligned}
\tag{4}
$$

We can now use the Einstein field equations to determine the stress-energy tensor $T_{\hat{\alpha}\hat{\beta}}$ that satisfies the given metric (neglecting the cosmological constant, G=Newton's constant).

$$G_{\hat{\alpha}\hat{\beta}} = 8\pi G T_{\hat{\alpha}\hat{\beta}} \tag{5}$$

However, Birkhoff's theorem [MTW, §32.2] allows only one vacuum solution to the Einstein field equations: the Schwarzschild solution. Therefore, $T_{\hat{\alpha}\hat{\beta}} \neq 0$ and combining Eqs. 4 and 5, we get [Visser, Eqs 11.36-38]:

$$T_{\hat{t}\hat{t}} = \rho = \frac{b'}{8\pi G r^2} \tag{6}$$

$$-T_{\hat{r}\hat{r}} = \tau = \frac{1}{8\pi G}\left[\frac{b}{r^3} - 2\left(1-\frac{b}{r}\right)\frac{\Phi'}{r}\right] \tag{7}$$

$$T_{\hat{\theta}\hat{\theta}} = T_{\hat{\varphi}\hat{\varphi}} = p = \frac{1}{8\pi G}\left\{\left(1-\frac{b}{r}\right)\left[\Phi'' + \Phi'\left(\Phi' + \frac{1}{r}\right)\right] - \frac{1}{2r^2}\left[b'r - b\right]\left(\Phi' + \frac{1}{r}\right)\right\} \tag{8}$$

Where ρ is energy density, τ is radial tension minus radial pressure, and p is transverse pressure. Combining Eqs. 6 and 7, we get:

$$8\pi G(\rho - \tau) = -\frac{e^{2\Phi}}{r}\left[e^{-2\Phi}\left(1-\frac{b}{r}\right)\right]' \tag{9}$$

Which leads to the inequality:

$$\rho(r_0) - \tau(r_0) \leq 0 \tag{10}$$

This specifies a negative mass-energy density at the wormhole's throat. Morris and Thorne named material with this property "exotic" [MT, p. 405]. Thus, in order not to have an impassable event horizon in our wormhole, we must thread the throat with exotic matter. This calls into question conditions 7 and 9. Morris and Thorne give several examples of wormholes that are well-behaved under conditions 1-6, and 8. We shall discuss the implications of exotic matter in the following sections.

In the most general case, the two asymptotically flat regions connected by the wormhole are not causally related, and we have an inter-universe wormhole. Specifying that the wormhole connect two regions in the same universe creates additional considerations [Visser, p. 110]:

1. Time can run at different rates between asymptotically flat regions. For an intra-universe wormhole, this corresponds to a nonconservative gravitational field. If you wait long enough, you will get a time machine. [Visser, pp. 239-240]

2. The tunnel joining the asymptotically flat regions could have a twist, like a Moebius strip. This creates a non-orientable spacetime. However, weak interactions are chiral, and chiral fermion fields only exist in orientable spacetime. Thus, non-orientable spacetime is incompatible with the standard model of particle physics. [Visser, pp. 286-287]

3. Energy Conditions

Wormhole construction depends crucially upon the existence of exotic matter. In this section, we will examine the feasibility of a negative stress-energy tensor by considering the various energy conditions that apply to general relativity. Paralleling Visser [Visser, §12], we use the Hawking-Ellis type I stress-energy tensor in an orthonormal frame:

$$T^{\hat{\alpha}\hat{\beta}} = \begin{bmatrix} \rho & 0 & 0 & 0 \\ 0 & p_1 & 0 & 0 \\ 0 & 0 & p_2 & 0 \\ 0 & 0 & 0 & p_3 \end{bmatrix} \tag{11}$$

The null energy condition (NEC) specifies that for any null vector:

$$T_{\alpha\beta}\, k^{\alpha} k^{\beta} \geq 0 \Leftrightarrow \forall j, \rho + p_j \geq 0 \tag{12}$$

The weak energy condition (WEC) specifies that for any timelike vector (V's are velocity vectors):

$$T_{\alpha\beta}V^{\alpha}V^{\beta} \geq 0 \Leftrightarrow \rho \geq 0 \wedge \forall j, \rho + p_j \geq 0 \tag{13}$$

The strong energy condition (SEC) specifies that for any timelike vector:

$$\left(T_{\alpha\beta} - \frac{T}{2} g_{\alpha\beta}\right)V^{\alpha}V^{\beta} \geq 0 \Leftrightarrow \forall j, \rho + p_j \geq 0 \wedge \rho + \sum_j p_j \geq 0 \tag{14}$$

Where T is the trace of the stress-energy tensor, $T = T_{\alpha\beta} g^{\alpha\beta}$.

The dominant energy condition (DEC) specifies that for any timelike vector:

$$T_{\alpha\beta}V^{\alpha}V^{\beta} \geq 0 \Leftrightarrow \rho \geq 0 \wedge \forall j, p_j \in [-\rho, +\rho] \tag{15}$$

This translates to the assertion that locally measured energy density is always positive, and energy flux is timelike or null. Note that the DEC implies the WEC, and the WEC implies the NEC.

The averaged null energy condition (ANEC) specifies that on a null curve Γ:

$$\int_{\Gamma} T_{\alpha\beta}k^{\alpha}k^{\beta} d\lambda \geq 0 \Leftrightarrow \int_{\Gamma}\left(\rho + \sum_j \cos^2 \psi_j p_j\right)\xi^2 d\lambda \geq 0 \tag{16}$$

Where λ is the generalized affine parameter of Γ, k^{α} is the tangent vector, $\cos\Psi_i$ are direction cosines, and:

$$k^{\hat{\alpha}} \equiv \xi(1; \cos\psi_i) \tag{17}$$

The averaged weak energy condition (AWEC) specifies that on a timelike curve Γ:

$$\int_{\Gamma} T_{\alpha\beta}V^{\alpha}V^{\beta} ds \geq 0 \Leftrightarrow \int_{\Gamma} \gamma^2 \left(\rho + \beta^2 \sum_j \cos^2 \psi_j p_j\right) ds \geq 0 \tag{18}$$

Where s is the proper time parameter of Γ, γ and β are from Special Relativity, V^{α} is the tangent vector, and:

$$V^{\hat{\alpha}} \equiv \gamma(1; \beta\cos\psi_i) \tag{19}$$

The averaged strong energy condition (ASEC) specifies that on a timelike curve Γ:

$$\int_{\Gamma}\left(T_{\alpha\beta}V^{\alpha}V^{\beta} + \frac{1}{2}T\right) ds \geq 0 \Leftrightarrow \int_{\Gamma}\left[\gamma^2\left(\rho + \beta^2 \sum_j \cos^2 \psi_j p_j\right) - \frac{1}{2}\rho + \frac{1}{2}\sum_j p_j\right] ds \geq 0 \tag{20}$$

For null vectors β→1, γ→∞ while γds→dλ, ds→0, and the ASEC reduces to the ANEC (to a multiplicative factor). AWEC and ASEC are included for completeness; Tipler has shown that ASEC violation implies singularities can be prevented by quantum mechanical effects[7].

We see from Eq. 10 and an inspection of Eqs. 12-15 that exotic matter violates the NEC, and correspondingly, the WEC, SEC, and DEC. Krasnikov[8] argues that this also implies violation of the ANEC, although he changes MT criterion 3 to "increasingly flat" which allows him to relax other restrictions, as discussed in the next section.

How physical is this? It is known from explicit calculations that the horizon of a Schwarzschild black hole also violates NEC, WEC, SEC, and DEC [Visser, §12.3.6]. Quantum field theory also violates these conditions, and in a final note to their paper, Wald and Yurtsever[9] demonstrate that the ANEC cannot hold in a general curved four-dimensional spacetime.

Energy considerations lead to topological considerations, in the following manner: The principle of topological censorship[10] generally encapsulates the idea that we cannot probe behind an event horizon:

"If an asymptotically flat, globally hyperbolic spacetime (M, g_{ab}) satisfies the averaged null energy condition, then every causal curve from J^- to J^+ deformable to γ_0 rel J." [FSW, p. 1486]

In particular, if the ANEC holds globally, then there cannot be a null geodesic through nontrivial topologies. Thus, topological censorship bars the existence of TLWs, and in fact, the converse of the theorem can be used to define a traversable wormhole. Therefore, the ANEC must be violated in order to have a TLW.

Taking these results together, the existence of "exotic matter" and TLW is not ruled out by energy conditions or topology.

4. Engineering Considerations

Morris and Thorne's 1988 paper demonstrated several examples "well-behaved" wormholes with reasonable tidal forces < 1g and transit times of < 1 year. From the previous sections, we know that exotic matter is required to banish the event horizon, and that there is nothing fundamental in physics which prevents the existence of exotic matter. In this section, we will discuss how much exotic matter is required, and where to get it.

Visser, Kar, and Dadhich[11] have demonstrated, using the metric of Eq. 1 and Schwarzschild geometry, that the volume of ANEC-violating matter can be given by:

$$\oint [p_r + 2p_t]dV = \frac{2\varepsilon m}{\sqrt{1-2\,m/a}} \qquad (21)$$

Where m is the mass of the wormhole, a is the radius of the region containing ANEC-violating matter (from r_0), b(r)=2m, and:

$$e^{2\Phi} = \varepsilon + \lambda\sqrt{1-2\,m/r} \qquad (22)$$

Thus, by suitable juggling of ε and a, the amount of exotic matter can be made arbitrarily small.

Sushkov[12] suggested looking for a wormhole with T_{ij}^{Q}, the quantum field operator, as the source for the metric given in Eq. 1. D. Hochberg, A. Popov, and S. Sushkov[13] found a possible solution, but the throat was Planck-sized. Subsequently, Krasnikov extended this work to include:

"A class of static Lorentzian wormholes with arbitrarily wide throats is presented in which the source of the WEC violations required by the Einstein equations is the vacuum stress-energy of the neutrino, electromagnetic, or massless scalar field." [Krasnikov, Abstract]

Krasnikov's work makes a number of important distinctions:

1. The asymptotic flatness of MT condition 3 is relaxed to "increasingly flat". This also has the effect of sidestepping the topological censorship theorem, since it requires asymptotic flatness.

2. Increasingly flat space should also be "increasingly empty".

Using the metric [Krasnikov, §2]:

$$ds^2 = \Omega^2(\xi)\left\{-d\tau^2 + d\xi^2 + K^2(\xi)[d\theta^2 + \sin^2\theta d\varphi^2]\right\} \tag{23}$$

Where Ω and K are smooth, positive, even functions, and considering the conditions above, Krasnikov converts the solution of the metric into a mathematical proof of the existence of $\Omega(\xi)$ and $K(\xi)$ with the desired properties. The proof of existence establishes that a TLW can be constructed from quantum effects.

We can therefore construct a TLW using an arbitrarily small amount of exotic matter obtained from the vacuum stress energy of neutrino, electromagnetic, or massless scalar fields.

5. Time Machines

Once a wormhole has been constructed, it can be converted into a time machine by simply accelerating one mouth of the wormhole with respect to the other [MTY, p. 1447]. Alternately, a time machine will result if two connected regions have a different rate of time flow.

The topic of time travel and its attendant paradoxes is an extremely wide-ranging one. For the purposes of this paper, time machines will be considered with respect to their possible effects on the existence of TLWs. To be precise, we employ the following [Visser, §17.1, Definition 17]:

"If a spacetime M contains a closed chronological curve (that is, a closed timelike curve) γ, then M contains a chronology-violating time machine, and the curve γ traverses the time machine."

Kim and Thorne[14] note that the boundary between spacetime without closed timelike curves [CTC] and spacetime that has CTCs established (by a TLW, in this case) is a Cauchy horizon. If the Cauchy horizon is unstable against perturbations such as those caused by fields within the spacetime, the creation of CTCs might be prevented. Furthermore, a TLW has certain null geodesics that will produce self-reinforcing vacuum fluctuations. However, Kim and Thorne concluded that the TLW divergent lens effect[15] on null geodesics coupled with quantum gravity cuts off vacuum-polarization divergence near the Cauchy horizon before it can prevent the creation of CTCs.

Opposition to the idea of time travel has been codified by Hawking's chronology protection conjecture[16]. Hawking suggests, contrary to Kim and Thorne, that quantum gravity cutoff does not involve observer-dependent time. Using invariant distance to the Cauchy horizon, he argues that vacuum fluctuations do indeed close the Cauchy horizon, preventing the creation of CTC. On the other hand, Hawking argues that negative stress-energy of a TLW has a repulsive gravitational effect, which prevents the creation of the Cauchy horizon in the first place. In general, Hawking postulates: "The laws of physics prevent the appearance of closed timelike curves." [Hawking, p. 610]

Krasnikov states that the essential problem is with the cosmic censorship hypothesis[17].

"To predict the outcome of (almost) any experiment we have to assume that our spacetime is globally hyperbolic. The wormholes, if they exist, cast doubt on the validity of this assumption. At the same time, no evidence has been found so far (either observational, or theoretical) that the possibility of their existence can be safely neglected." [KrasnikovII, Abstract]

The creation of closed timelike curves and the abolition of global hyperbolicity are two possible causes that may prevent the creation of traversable Lorentzian wormholes. However, despite the potential loss of causality in physics, it is still far from certain that TLWs cannot exist.

5. Conclusions

General relativity does not preclude the existence of traversable Lorentzian wormholes. A Lorentzian wormhole can be engineered to be quickly and safely traversable, provided the throat of the wormhole is threaded with exotic matter. Considering quantum field theory, general relativity, and topology, we have found that exotic matter is not ruled out by energy conditions. With fine-tuning, the amount of exotic matter required can be made arbitrarily small, and sufficient amounts can be obtained from vacuum stress energy to create a traversable Lorentzian wormhole with an arbitrarily large-sized throat. This contrasts with other proposed forms of faster than light travel, such as warp drives[18], which require exotic energy amounts orders of magnitude greater than the mass of the visible universe[19]. Finally, the existence of traversable Lorentzian wormholes implies the existence of time machines and the loss of global hyperbolicity, which casts the ultimate predictability of theories of physics in grave doubt. Yet, no evidence so far presented can distinguish between the following four hypotheses [Visser, Chapter 19]:

1. The radical rewrite conjecture – Allow non-Hausdorff manifolds, forbid causality
2. The Novikov consistency conjecture – Allow closed timelike curves, forbid inconsistencies
3. The chronology protection conjecture – Allow traversable wormholes, forbid closed timelike curves
4. The boring physics conjecture – Forbid all physical mechanisms allowing closed timelike curves, such as traversable wormholes

Ultimately, the quest to delineate which of the four hypotheses is true has both fulfilled Morris and Thorne's intended goal of using it as a pedagogical exercise in general relativity, and allowed us to continue the dream of one day reaching the stars.

References

[1] A. Einstein and N. Rosen. “The particle problem in the generalized theory of relativity.” Phys. Rev., 48:73-77, 1935

[2] J. Wheeler. “Geons.” Phys. Rev., 97:511-536, 1955

[3] M. Morris and K. Thorne. “Wormholes in spacetime and their use for interstellar travel: A tool for teaching general relativity.” Am. J. Phys., 56:395-412, 1988, cited in text as MT

[4] M. Morris, K. Thorne, and U. Yurtsever. “Wormholes, time machines, and the weak energy condition.” Phys. Rev. Lett., 61:1446-1449, 1988

[5] M. Visser, Lorentzian wormholes — from Einstein to Hawking, AIP Press, New York, 1995, cited in text as Visser.

[6] C. Misner, K. Thorne, and J. Wheeler. Gravitation, Freeman, New York, 1973, cited in text as MTW.

[7] F. Tipler. “Energy conditions and spacetime singularities.” Phys. Rev. D 17:2521-8, 1978

[8] S. Krasnikov. “A traversable wormhole.” Phys. Rev. D 62, 2000, arXiv:gr-qc/9909016, cited in text as Krasnikov.

[9] R. Wald and U. Yurtsever. “General proof of the averaged null energy condition for a massless scalar field in two-dimensional curved spacetime”. Phys. Rev. D 44:403-416, 1991

[10] J. Freidman, K. Schleich, and D. Witt. “Topological Censorship.” Phys. Rev. Lett., 71:1486-1489, 1993, cited in text as FSW.

[11] M. Visser, S. Kar, and N. Dadhich. “Traversable wormholes with arbitrarily small energy condition violations.” Phys. Rev. Lett., 90:201102-1:4, 2003, arXiv:gr-qc/0301003

[12] S. V. Sushkov, Phys. Lett. A 164, 33, 1992

[13] D. Hochberg, A. Popov, and S. Sushkov,. “Self-Consistent Wormhole Solutions of Semiclassical Gravity.” Phys. Rev. Lett. 78:2050-3, 1997

[14] S. Kim and K. Thorne. “Do vacuum fluctuations prevent the creation of closed timelike curves?”. Phys. Rev. D., Vol. 43, Number 12, pp. 3929-3947, 1991.

[15] J. Cramer, R. Forward, M. Morris, M. Visser, G. Benford, and G. Landis. “Natural Wormholes as Gravitational Lenses.” Phys. Rev. D51, 3117-3120, 1995, arXiv:astro-ph/9409051

[16] S. Hawking. “Chronology protection conjecture.” Phys. Rev. D, Vol. 46, Number 2, pp. 603-11, 1992, cited in text as Hawking.

[17] S. Krasnikov. “The wormhole hazard”. Talk given at “Time and Matter”, Venice, 2002, arXiv:gr-qc/0302107, cited in text as KrasnikovII.

[18] M. Alcubierre. “The warp drive: hyper-fast travel within general relativity.” Class. Quant. Grv. 11, L73-L77, 1994, arXiv:gr-qc/0009013

[19] M. Pfenning. “Quantum Inequality Restrictions on Negative Energy Densities in Curved Spacetimes.” PhD. Dissertation, Tufts University, Medford, Massachusetts, 1998, arXiv:gr-qc/9805037